ASSIGNMENT SET - I

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

B.Sc Hon.(CBCS)

Mathematics: Semester-II

Paper Code: C3T

[REAL ANALYSIS]

Answer all the questions

1. Construct a bounded set of real numbers with exactly three limit points.
2. Is the sequence $\{(-1)^n/n\}$ a Cauchy sequence ? Justify your answer.
3. Give an example of an infinite series $\sum_{n=1}^{\infty} a_n$ such that $(a_1 + a_2) + (a_3 + a_4) + \ldots$ converges but $a_1 + a_2 + a_3 + a_4 + \ldots$ diverges.
4. Determine whether the sequence $\{-2n + \sqrt{(4n^2 + n)}\}$ is a Cauchy sequence or not.
5. If $\sum_{n=1}^{\infty} a_n$ converges then prove that $\lim\limits_{n=\infty} a_n = 0$. Is the converse true ? Justify
6. Find supA and Inf A, where $A = \{x \in R : 3x^2 + 8x - 3 < 0\}$
7. Show that the set of all even integers is not compact .
8. If p>0 and it is a real number, then find the limit of the sequence $\{\frac{n^t}{((1+p)^n)}\}$
9. If y is a positive real number then show that there exist a natural number m such that $0 < \frac{1}{2^m} < y$.
10. Find the derived set of the set **S** $= \{(-1)^n (1 + \frac{1}{n}) : n \in N\}$.
11. Show that the sequence $\{(1 - \frac{1}{n})\cos(\frac{n\pi}{2})\}$ is not convergent , but has a convergent subsequence .
12. State Archimedean property of real number and hence show that $\lim\limits_{n \to \infty} \frac{x}{y} = 0$
13. Verify that the series $\sum_{n=1}^{\infty} sin\frac{1}{n}$ is not convergent.

14. Construct an unbounded sequence with exactly one subsequential limit .
15. If $\{s_n\}$ is a sequence of real number and if $s_n \leq M$ for all n∈ N, and if $\lim_{n\to\infty} s_n$ = L, then prove that L ≤M.
16. For any two real numbers a,b with a < b, prove that there exist a rational number r such that a < r < b.
17. Find the limit superior and limit interior of the sequence $\{ 1 + (-1)^n + \frac{1}{2^n} \}$
18. Prove that every bounded decreasing sequence is convergent.
19.
    i) If $\sum_{n=1}^{\infty} a_n$ diverges then prove that $\sum_{n=1}^{\infty} n a_n$ also diverges.
    (ii) Let A and B be two subsets of R. If int A = int B =∅ and if A is closed in R, then find int (A∪B).
20. For any sequence $\{a_n\}$ of positive real numbers, prove that $\lim_{n\to\infty} \inf \frac{a_{n+1}}{a_n} \leq \lim_{n\to\infty} \sqrt[n]{a_n}$.
21.
    i) If p is a limit point of a subset S of real number s, then prove that there exists a countably infinite subset of S having p as its only limit point .
    (ii)Let S be a non- empty subset of real number s which is bounded below and T = { -X: x∈ S } . Prove that the set T is bounded above and Sup T = - inf S
22. (i) Let A be subset of R . One of the following statements is true and the other is false . Identify the true statement and prove it . Identify the false statement with proper arguments
    A. very interior point of A is a limit point of A .
    B. very limit point of A is an interior Point of A
23. Examine the convergence of the sequence $\{x_n\}$ where $x_n = \sum_{k=1}^{n} \frac{3k^2+2k}{2^k}$
24. i) If θ is a rational number , then examine whether the sequence { sin($n\theta\pi$) } has a limit .
    (ii) If $\sum_{n=1}^{\infty} a_n$ is a convergent then test the convergence of the series $\sum_{n=1}^{\infty} \frac{a_n}{\log(n+1)}$.
25. (i) Let S and T be two nonempty bounded subset of R such that S is a subset of T. Prove that inf T≤ inf S .
    (ii) Test for the convergence of the series $\frac{3}{5}x^2 + \frac{4}{5}x^3 + \frac{15}{17}x^4 + \frac{12}{13}x^5 + \ldots$,x > 0
26.
27. (i) Let S be a sequence of real numbers . Show that every subsequence of a subsequence of S is itself a subsequence of S.
    (ii) Let a sequence of positive real numbers $\{x_n\}$ converge to x. Prove that the sequence $\{\sqrt{x_n}\}$ converges to $\sqrt{x}$.

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